

Working Paper: WP 109

# A Dynamic Stochastic General Equilibrium Model for India



# A Dynamic Stochastic General Equilibrium Model for India

# NCAER Working Paper

Shesadri Banerjee and Parantap Basu\*

May 21, 2015

## Abstract

Over the last decade, the Dynamic Stochastic General Equilibrium (DSGE) framework has become a workhorse for macroeconomic analysis in both academic and policy circles. Following this emerging trend, we aim to expand our research capacity in macroeconomics at NCAER by introducing a baseline DSGE model for the Indian economy. This working paper comes out as a part of this process. In this paper, we make two contributions. First, we explore the empirical regularities of the Indian business cycle and establish a few stylized facts. Second, we produce a baseline DSGE model that can serve as an analytical framework for understanding these stylized facts. The model has a small open economy feature with a clear demarcation between consumption and investment goods sectors. We simulate the model with plausible parameterization based on the DSGE literature. Our results show that the baseline model can replicate the stylized facts reasonably well.

**Keywords**: *DSGE Modeling, IST and TFP Shocks* **JEL Classification**: *E2, E6.* 

\*Shesadri Banerjee is an Associate Fellow at NCAER, and specializing in the Dynamic Stochastic General Equilibrium (DSGE) modeling. Parantap Basu is a Professor of Macroeconomics and Director of the Centre for Economic Growth and Policy at Durham University Business School, UK. Please direct your comments and suggestions to <u>sbanerjee@ncaer.org</u>

@This working paper has come out from the project entitled with "Dynamic Stochastic General Equilibrium Modelling for the Indian Economy" sponsored by the TTI, IDRC grant.

**Disclaimer:** The NCAER Working Paper Series has been developed in the interest of disseminating on-going work undertaken by NCAER sta¤ and consultants in a timely fashion. Research presented here is work-in-progress that has not received external scrutiny and may be less than fully polished. The papers carry the names of the authors and should be cited accordingly. The .ndings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the National Council of Applied Economic Research or those its Governing Body members.

## 1 Introduction

Maintaining a positive macroeconomic environment to sustain high income growth with stability is an important policy target, particularly for emerging market and developing economies like India. A policy framework that addresses concerns relating to fiscal and external imbalances, inflation, and growth must take into account the increasingly complex, uncertain and interlinked global economy. India has set out to modernise its macroeconomic policy apparatus, particularly in the area of monetary policy, by bringing about greater transparency on its goals and instruments.<sup>1</sup> Possessing research capacity to conduct and analyse policy simulations that explore alternative scenarios and assist in policy development is, therefore, a crucial step in moving towards a more dynamic policy environment. In this regard, a well specified and operational Dynamic Stochastic General Equilibrium (DSGE) model for the Indian economy can be utilised to explain, analyse, and forecast the cyclical properties of macroeconomic aggregates. Recognising the growing need for modern policy analysis tools, National Council of Applied Economic Research (NCAER) undertakes a research initiative to develop a DSGE model for India on an accelerated basis. This working paper is prepared as an initial step of that research initiative. In the backdrop of current DSGE literature for India, the paper aims to present a Small Open Economy (SOE) DSGE model for the Indian economy and endeavours to replicate the key stylised facts of the Indian economy business cycles as observed from the data.

An intriguing stylised macroeconomic fact for the Indian economy is that the cyclical components of GDP and inflation negatively correlate. Such regularity is robust to the filtering procedure to extract the cyclical components from the data. This stylised fact is intriguing because it goes contrary to the conventional Phillips curve trade off between short run output gap and inflation. While a new breed of recent literature is emerging on modelling Indian macroeconomy using DSGE framework, none of these papers shed light on this apparent empirical puzzle. Apart from the countercyclical movement of inflation, procyclical movement of consumption and investment are observed from the data. Further, it is also noticed that investment and depreciation of the nominal exchange rate are positively correlated to short term nominal interest rate.

<sup>&</sup>lt;sup>1</sup>Discussions with the researchers from the Reserve Bank of India, Mumbai, and the Department of Economic Affairs in the Ministry of Finance also suggest their growing interest in such rigorous policy analysis tools.

Based on these stylised facts, we develop a baseline DSGE model in this paper with two primary objectives. First, we propose to shed some lights on the empirical anomaly regarding the short run relationship between inflation and output gap. Second, we propose to develop a baseline DSGE model which could provide a framework for policy analysis and macroeconomic forecasting for India. None of the papers on DSGE modelling of India has set these two goals. At this initial stage, we have kept the model parsimonious intentionally. However, the model can be extended later to bring some additional key frictions which are endemic to the Indian economy. Our model has frictions such as staggered price setting, aggregate habit persistence, investment adjustment cost, and transaction costs of foreign bond holding which makes uncovered interest parity to fail. The model also allows for a combination of domestic and external shocks which include, shocks to total factor productivity (TFP), shocks to investment specific technology (IST), monetary and fiscal policy shocks and foreign demand and foreign interest rate shocks. The last two external shocks are typical features of a small open emerging economy like India.

Among the domestic shocks, a major redeeming feature of our baseline model analysis is that we find significant importance of IST shock as a major driver of output variation. None of the existing papers on Indian DSGE models highlight the importance of this IST shock. The IST shock is crucially important for Indian economy because it determines the relative price of investment. Parente and Prescott (2000) attach significant importance to this IST shock as a determinant of barriers to riches of nations. Countries may erect barriers to the use of efficient technology which could drive up the relative price of investment goods and could lower total factor productivity through this channel. The variance decomposition analysis based on our baseline DSGE model suggests that this IST explains about 60 percent of the variation of GDP in India.

There is another important dimension where our model differs from the extant Indian DSGE models. We distinguish between home and foreign produced consumption and investment based intermediate goods. This distinction is important because it makes the relative price of investment goods depend on the IST shock as well as the external terms of trade via the relative home biases in consumption and investment as in Basu and Thoenissen (2011). This flexibility allows us to explore the effect of IST shock on the aggregate dynamics via influencing the terms of trade. Allowing this feature makes the IST shock play a predominant role in our model.

In addition, our model has standard monetary policy shock which is basically a shock to the interest rate rule. We find that the impulse response paths of output and inflation following a monetary policy shock differs from the impulse responses of IST and TFP shocks. Since the correlation between output and inflation is a summary of the responses of output and inflation to each of these structural shocks, different impulse responses of output and inflation could provide more insights why inflation and output could be negatively correlated in our baseline model. In addition, our model is capable of reproducing two additional stylised facts of the Indian economy that both consumption and investment are procylical.

The rest of the paper is organised in the following way. Section Two will provide a brief review of the existing literature on DSGE modelling for India. Section Three will present the key stylised facts. Section Four will describe the model. Section Five will show the baseline calibration of the model. Section Six concludes the paper.

# 2 A Sketch of Literature on DSGE Modelling for India

While the practice of macroeconomic modelling has changed substantially both in the developed and developing countries over the last decade, application of the DSGE models is slowly receiving attention in the Indian context. Historically, the tradition of macroeconomic analysis and forecasting in India has rested on macro-econometric models and remained largely a-theoretic. Introducing structures that are more clearly specified in theory would add greater coherence to the models. A few pioneering studies on DSGE modelling for India have been reported very recently. Some of them are discussed below.

Anand et al., (2010) made the first attempt to produce a DSGE model for the Indian economy. Their study was essentially motivated to examine the role of balance sheets in the transmission of shocks to the Indian economy. They developed a small open economy model with financial accelerator where firms are able to borrow in both domestic and foreign currency. Financial accelerator mechanism was incorporated to study the effect of financial frictions on the real economy. Using Bayesian method of estimation, they estimated the macro-financial linkages for India. In light of such linkages, they analysed the conduct of monetary policy.

Levine and Pearlman (2010) used a DSGE model for India to analyse the 'three pillars macro-

economic policy framework' which was a combination of a freely floating exchange rate, an explicit target for inflation over the medium run, and a mechanism that ensures a stable government debt to GDP ratio around a specified long run. This study emerged in motivation to build up a macroeconomic framework that will be resilient to domestic and external shocks. In their model, the price of consumer goods depends on the exchange rate. The exporting firms typically set their prices in foreign currency and bear the risk of currency fluctuations. Exporting firms could borrow from international capital markets in foreign currency, so that debt repayment is similarly affected. Therefore, foreign shocks had significant effects on the domestic economy. Given this structure of the model, Levine and Pearlman (2010) developed an optimal rules based interpretation of the 'three pillars' and show how such monetary fiscal rules need to be adjusted to accommodate specific features of emerging market small open economies (SOEs). The salient point was a SOE like India faces substantially different policy issues from those of advanced, larger, and more closed economies. Thus, the monetary and fiscal policy prescriptions need to be fundamentally different from those in an advanced economy.

Gabriel et al., (2010) studied the business cycle dynamics of the Indian economy using a New Keynesian DSGE model. They developed a closed economy DSGE model and estimated that by Bayesian Maximum Likelihood estimation technique. Their model comprised by a formal and and informal sectors, and featured New Keynesian frictions in the form of imperfect competition, sticky price, investment adjustment cost, credit-constrained consumers, and the financial accelerator facing domestic firms seeking to finance their investment. They examined the properties of the simulated variables under generalized inflation targeting Taylor type interest rate rule with forward and backward looking components.

Goyal (2011) studied the effect of monetary policy shock in a New Keynesian DSGE model with dualistic labour markets. Comparing the structure of Small Open Emerging Market Economy (SOEME) and Small Open Economy (SOE) with respect to consumption inequality and the labour market, aggregate supply curve was derived and subsequently the implications of alternative monetary policies were envisaged. The paper showed that flexible domestic inflation targeting produces the lowest volatility although there is a trade-off. Consumer price targeting performs better when combined with some kind of managed floating. Bhattacharya and Pattnaiyk (2012) have also worked on the DSGE modelling for India addressing the business cycle features of consumption volatility.

Although, there is a growing interest among the researchers to adopt the DSGE framework, application of these models for the macroeconomic policy analysis, projections, and policy simulations has not yet become a part of the mainstream research. In addition, these models do not examine the relative importance of TFP and IST shocks in an open economy context via the terms of trade channel. Nor do they investigate why output - inflation relation is negative in India. To the best of our knowledge, our paper is the first step in that direction.

## 3 Stylised Facts of Indian Business Cycle

Literature shows that stylised facts come from the statistical properties of the broad regularities of the business cycles (Lucas, 1976). In this spirit, we have explored the cross correlations among the major macroeconomic variables over the cyclical fluctuations. Motivation behind this empirical exercise is to study the comovements of the variables as well as to examine the robustness of their relationships. In order to examine the comovements over the business cycle, first, we select five key macroeconomic variables, namely, consumption, investment, employment, inflation, and depreciation of nominal exchange rate. The reason for selecting these variables is that we find statistically significant bivariate correlations for which our model provides some insights. Next, we look at their relationship with the real output, real government spending, and short term nominal interest rate over the cyclical components. We use three filtering techniques which dominate the extant literature of business cycle. These are HP filter (1980), BK filter (1999), and CF filter (2003) band pass filter. These filtering procedures eliminate the nonlinear trends from the data and yield cyclical deviations. We use annual data over the sample period of 1970 to 2010. In Table 1, we present the cross correlation coefficients among the variables mentioned above for each of the filtering procedures and assign their statistical significance.<sup>2</sup>

Table 1 reveals the empirical features of the key Indian macroeconomic variables. We find procyclical movement of consumption and investment but countercyclical movement of inflation.

<sup>&</sup>lt;sup>2</sup>Sources of data on all macroeconomic variables are given in the Data Appendix. We assign '\*', '\*\*', and '\*\*\* ' for the statistical significance at the level of 10%, 5%, and 1%, respectively.

Business Cyclical Component using HP Filter							
	Output	Public Spending	Nominal Interest Rate				
Consumption	0.796***	0.460***	-0.104				
Investment	0.384**	-0.004	0.366**				
Employment	-0.047	0.463***	-0.130				
Inflation	-0.495***	-0.194	0.156				
Exchange rate Depriciation	-0.048	-0.004	0.446***				
Business Cyclical Component using BK Filter							
	Output	Public Spending	Nominal Interest Rate				
Consumption	0.871***	0.148	-0.092				
Investment	0.277*	-0.068	0.407**				
Employment	0.101	0.127	0.081				
Inflation	-0.601***	-0.223	0.190				
Exchange rate Depriciation	-0.075	-0.112	0.422**				
Business	Cyclical Con	nponent using CF I	Filter				
	Output	Public Spending	Nominal Interest Rate				
Consumption	0.870***	-0.008	0.101				
Investment	0.301**	-0.052	0.461***				
Employment	-0.219	0.181	-0.183				
Inflation	-0.708	-0.251	0.029				
Exchange rate Depriciation	0.007	-0.235	0.388**				

Table 1: Evidence on Co-movements from Cross-correlations (1970 - 2010)

While the comovement of the first two variables are plausible, it is surprising to see that inflation and output are negatively correlated. This stylized fact is surprising because it goes contrary to the conventional New Keynesian wisdom that there is a Phillips curve trade off between short run output and inflation. Although a new breed of recent literature is emerging on modelling Indian macroeoconomy using DSGE framework, none of these papers shed any light on this apparent empirical puzzle. This negative correlation between the cyclical components of inflation and output is robust to the filtering procedures of extracting the cyclical components from the data. Apart from this, it is also noticeable that investment and depreciation of the nominal exchange rate are positively correlated to nominal interest rate.

## 4 The Model

Our model environment is similar to Basu and Thoenissen (2011). We consider a small open economy with incomplete financial markets. Each country produces one tradable intermediate good that is used in the home and foreign consumption and investment goods baskets. Similar specification of consumption and investment goods is found in Heathcote and Perri (2002), Backus et al. (1994), Thoenissen (2008), and Basu and Thoenissen (2012). To address the business cycle features of the emerging market or developing economy like India, this model incorporates various frictions and shocks, as proposed in Kollmann (2002), Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005). We consider frictions in the form of external habit formation in consumption, investment adjustment costs, transaction cost of foreign bond holding and staggered price setting of the intermediate goods producing firms. The model dynamics are driven by six shocks, namely, total factor productivity (TFP), investment specific technology (IST), monetary and fiscal policy shocks, foreign interest rate and foreign demand shocks.

## 4.1 Description of the economy

Representative household owns the physical capital stock, supplies labour and rents capital to the intermediate goods firms. At date t, household's receives its proceeds from wage income, rental income, profit from the ownership of firms and interest income from domestic and foreign bond holding. The household uses its income at date t by consuming final consumption goods, investing in physical capital, and buying new bonds (domestic as well as foreign).

There are two kinds of firms, final goods and intermediate goods. Final goods firms produce two types of goods, namely, consumption goods and investment goods which are not internationally traded. On the other hand, intermediate goods firms produce goods which can be used for processing consumption and investment goods and these intermediate goods are also tradeable. We assume that these intermediate goods firms produce differentiated variety of goods and as a result each producer has some monopoly power of price setting. The nexus of final and intermediate goods producing firms resembles to Kollmann (2002) and Basu & Thoenissen (2009).

There is a government in charge of fiscal spending. Government spending is in the form of final goods consumption and is financed by the taxes and domestic borrowing.

The Central Bank follows a Taylor type interest rate to respond to inflation and business cycle conditions.

Figure 1 summarises the interaction among all the economic agents in the model.



Figure 1: Structure of the Model

## 4.2 Representative household

There are continuum of agents in the home economy in unit interval. The j th home-consumer is maximising the following present value of its lifetime expected utility subject to standard flow budget constraints.

$$E_0 \sum_{t=0}^{\infty} \beta^t V\left[ (C_t^j - \gamma_c C_{t-1}), L_t^j) \right]$$

$$\tag{1}$$

where  $E_0$  denotes the conditional expectation at date t,  $\beta$  is the intertemporal discount factor, with  $0 < \beta < 1$ . There is aggregate habit formation which means that the consumer receives utility from current consumption,  $C_t^j$  after adjusting for the previous period's aggregate level of consumption,  $C_{t-1}$  and suffers disutility from supplying labour,  $L_t^j$ . Utility function is additively separable in consumption and labour, and is specified as follows:

$$V(.) = \left[\frac{1}{1 - \sigma_c} \left(C_t^j - \gamma_c C_{t-1}\right)^{1 - \sigma_c} - \frac{1}{1 + \sigma_l} \left(L_t^j\right)^{1 + \sigma_l}\right]$$
(2)

where  $\sigma_c$  denotes the intertemporal elasticity of substitution in consumption and  $\sigma_l$  is the inverse of Frisch elasticity of labour.

Home residents trade two nominal riskless bonds of one period maturity denominated in the domestic and foreign currency respectively. Residents in both countries issue these bonds to finance their consumption expenditures. We follow Benigno (2009) by assuming that home bonds are only traded nationally while foreign residents can allocate their wealth in foreign bonds denominated in the foreign currency. This asymmetry in the financial market structure is reflecting the stark nature of capital control facing a developing country like India. The international financial market is thus incomplete because only a riskless foreign currency deaminated bond is internationally traded. There is a transaction cost facing the home households when they take a position in the foreign bond market. As in Benigno (2009), this cost depends on the net foreign asset position of the home economy.

The household also purchases investment goods  $(X_t^j)$  at a price  $P_{x,t}$  to undertake capital accumulation using the investment technology:

$$K_{t+1}^{j} = (1-\delta)K_{t}^{j} + [1 - S(X_{t}^{j}/X_{t-1}^{j})]X_{t}^{j}$$
(3)

where  $\delta$  is the rate of depreciation of the capital stock and S(.) captures investment adjustment costs as proposed by Christiano et al. (2005). We make standard assumption that S(1) = S'(1) = 0and  $S''(1) = \varkappa > 0$ . The implication is that adjustment cost disappears in the long run.

The representative home consumer thus faces the following budget constraint:

$$P_t C_t^j + P_{x,t} X_t^j + \frac{B_{H,t}^j}{(1+i_t)} + \frac{\xi_t B_{F,t}^j}{(1+i_t^*) \Theta\left(\frac{\xi_t B_{F,t}^j}{P_t}\right)} = W_t L_t^j + R_{k,t} K_t^j + B_{H,t-1}^j + \xi_t B_{F,t-1}^j + \Omega_t^{d,j} - T_t \quad (4)$$

where  $B_{H,t}^{j}$  and  $B_{F,t}^{j}$  are the individual's holdings of domestic and foreign nominal riskless bonds denominated in the local currency,  $i_{t}$  is the home country nominal interest rate,  $i_{t}^{*}$  is the foreign country nominal interest rate,  $\xi_t$  is the nominal exchange rate expressed as the price of one unit of foreign currency in terms of home currency,  $P_t$  is the consumer price level and  $W_t$  is the nominal wage. The representative home household supplies labour and rents capital to the domestic intermediate goods firms which explains the remaining wage and rental income terms,  $W_t L_t^j$ ,  $R_{k,t} K_t^j$ in the household's flow budget constraint. In addition,  $\Omega_t^{d,j}$  is the profit income of the household from the domestic intermediate goods producing firms. Home agents own all domestic intermediate firms and the equity holding within these firms is evenly divided between domestic agents.<sup>3</sup>

The cost function  $\Theta(.)$  drives a wedge between the returns on foreign and home bonds. Benigno (2009) ascribes this cost to the existence of foreign-owned intermediaries in the foreign asset market who apply a spread over the risk-free rate of interest when borrowing or lending takes place to home agents in foreign currency. The implication is that the home country borrows from the foreign country at a premium but lends at a discount. The spread between the borrowing and lending rates depends on the net foreign asset position of the home economy. Profits from this activity in the foreign asset market are distributed equally among foreign residents. In the steady state this spread is zero. The cost function  $\Theta(.)$  is unity only when the net foreign asset position is at its steady state level, i.e.  $B_{F,t} = \overline{B}$ , and is a differentiable decreasing function in the neighbourhood of  $\overline{B}$ .

Household's first order conditions are

$$C_t^j : (C_t^j - \gamma_c C_{t-1})^{-\sigma_c} - \lambda_t P_t = 0$$

$$\tag{5}$$

$$L_t^j : -L_t^{j\sigma_L} + \lambda_t P_t(W_t/P_t) = 0 \tag{6}$$

$$K_{t+1}^{j}: -\mu_{t} + E_{t}\mu_{t+1}(1-\delta) + E_{t}\lambda_{t+1}P_{t+1}(R_{k,t+1}/P_{t+1}) = 0$$
(7)

$$X_{t}^{j}: \mu_{t} \left[ \left(1 - s(X_{t}^{j}/X_{t-1}^{j})\right) - s'(X_{t}^{j}/X_{t-1}^{j})(X_{t}^{j}/X_{t-1}^{j}) \right] + E_{t}\mu_{t+1}s'(X_{t+1}^{j}/X_{t}^{j})(X_{t+1}^{j}/X_{t}^{j})^{2} - \lambda_{t}P_{t}(P_{x,t}/P_{t}) = 0$$

$$\tag{8}$$

$$B_{H,t+1}^{j}: -\lambda_t \cdot \frac{1}{1+i_t} + E_t \lambda_{t+1} = 0$$
(9)

 $<sup>^{3}</sup>$ Note that positive profit arises from the ownership of monopolistic intermediate goods firms only because retail firms are all competitive.

$$B_{Ft+1}^{j} : \frac{-\xi_{t}\lambda_{t}}{(1+i_{t}^{*})\Theta\left(\frac{\xi_{t}B_{F,t}^{j}}{P_{t}}\right)} + E_{t}\xi_{t+1}\lambda_{t+1} = 0$$
(10)

where  $\lambda_t$  and  $\mu_t$  are the Lagrangian multipliers associated with the nominal flow budget constraint (4), and the capital accumulation technology (3) respectively.

Next, note that the Tobin's q (the opportunity cost of investment in terms of foregoing consumption) is defined as:

$$q_t = \frac{\mu_t}{\lambda_t P_t}$$

Using this definition of q the Euler equation (8) can be rewritten as:

$$q_t \left[ (1 - s(X_t^j / X_{t-1}^j)) - s'(X_t^j / X_{t-1}^j)(X_t^j / X_{t-1}^j) \right] + E_t q_{t+1} s'(X_{t+1}^j / X_t^j)(X_{t+1}^j / X_t^j)^2 m_{t+1} = P_{x,t} / P_t$$

where  $m_{t+1}$  is the stochastic discount factor and expressed as:  $m_{t+1} = \beta (C_{t+1}^j - \gamma_c C_t)^{-\sigma_c} / (C_t^j - \gamma_c C_{t-1})^{-\sigma_c}$ 

So, equation (7) can be written as:

$$q_t = E_t q_{t+1} (1 - \delta) m_{t+1} + E_t m_{t+1} (R_{k,t+1}/P_{t+1})$$
(11)

As in Benigno (2009), all individuals belonging to the same country have the same level of initial wealth. This assumption, along with the fact that all individuals face the same labour demand and own an equal share of all firms, implies that within the same country all individuals face the same budget constraint. Thus they will choose identical paths for consumption. For this reason of symmetry, hereafter we drop the suffix j.

## 4.3 Final goods producing firms

## 4.3.1 Consumption goods sector

There are competitive distributors who package home and foreign intermediate consumption goods  $(C_{H,t} \text{ and } C_{F,t})$  to deliver final consumption goods  $(C_t)$  to the household. While packaging these

two goods, they use the following CES technology.

$$C_t = \left[ v^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$
(12)

where  $\theta$  is the elasticity of intra-temporal elasticity of substitution between home and foreignproduced final consumption goods and v is the home bias in consumption.

Both home and foreign consumption goods consist of a continuum of intermediate goods in the unit interval based on the following CES technology:

$$C_{H,t} = \left[ \int_0^1 C_{H,t}^{\frac{\varepsilon-1}{\varepsilon}}(i) di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$
(13)

$$C_{F,t} = \left[ \int_0^1 C_{F,t}^{\frac{\varepsilon-1}{\varepsilon}}(i) di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$
(14)

Cost minimisation by final consumption goods producers yields the following input demand functions for the home economy (similar conditions hold for foreign producers).

$$C_{H,t}(i) = v \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t$$

$$C_{Ft}(i) = (1-v) \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t$$
(15)

The consumer price index (CPI) that corresponds to the previous demand function is defined as:

$$P_t = [vP_{H,t}^{1-\theta} + (1-v)P_{F,t}^{1-\theta}]^{1/(1-\theta)}$$
(16)

while

$$P_{H,t} = \left[\int_0^1 P_{H,t}^{1-\varepsilon}(i)di\right]^{\frac{1}{1-\varepsilon}}$$
(17)

and

$$P_{F,t} = \left[\int_0^1 P_{F,t}^{1-\varepsilon}(i)di\right]^{\frac{1}{1-\varepsilon}}$$
(18)

 $P_{H,t}$  and  $P_{F,t}$  will be determined by price setting behaviour of domestic intermediate goods pro-

ducing firms and foreign owned intermediate goods importing firms which will be specified later.

## 4.3.2 Investment goods sector

Final investment goods  $(X_t)$  are produced by combining home and foreign-produced intermediate goods  $(X_{H,t} \text{ and } X_{F,t})$  in an analogous manner:

$$X_{t} = Z_{x,t} \left[ \varphi^{\frac{1}{\tau}} X_{H,t}^{\frac{\tau-1}{\tau}} + (1-\varphi)^{\frac{1}{\tau}} X_{F,t}^{\frac{\tau-1}{\tau}} \right]^{\frac{\tau}{\tau-1}}$$
(19)

where  $\varphi$  is the home bias in investment and  $\tau$  is the elasticity of substitution between home and foreign intermediate inputs and  $Z_{x,t}$  is investment specific technology shock (IST) and it appears in the investment goods production function as a total factor productivity (TFP) term as in Basu and Thoenissen (2011).

$$X_{H,t} = \left[\int_0^1 X_{H,t}^{\frac{\varepsilon-1}{\varepsilon}}(i)di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$
(20)

$$X_{F,t} = \left[\int_0^1 X_{F,t}^{\frac{\varepsilon-1}{\varepsilon}}(i)di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$
(21)

Cost minimisation by these investment goods firms yields the following demand functions:

$$X_{H,t}(i) = \varphi \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{x,t}}\right)^{-\tau} X_t$$
(22)

$$X_{F,t}(i) = (1 - \varphi) \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_{x,t}}\right)^{-\tau} X_t$$
(23)

where the investment goods price index (or the producer price index, PPI) is given by:

$$P_{x,t} = \left[\varphi P_{H,t}^{1-\tau} + (1-\varphi) P_{F,t}^{1-\tau}\right]^{1/(1-\tau)}$$
(24)

The PPI is a function of the price of home and foreign-produced intermediate goods prices. It differs from the CPI due to different substitution elasticities, different degrees of consumption and investment home biases.

### 4.3.3 Completing the price nexus

The price indices for consumption and investment goods are given by:

$$P_{t} = P_{H,t} \left[ \nu + (1 - \nu) (P_{F,t}/P_{H,t})^{1-\theta} \right]^{1/(1-\theta)}$$

$$P_{x,t} = P_{H,t} \left[ \varphi + (1 - \varphi) (P_{F,t}/P_{H,t})^{1-\tau} \right]^{1/(1-\tau)} (1/Z_{x,t})$$
(25)

Thus, the relative price of investment is:

$$\frac{P_{x,t}}{P_t} = \frac{\left[\varphi + (1-\varphi)(P_{F,t}/P_{H,t})^{1-\tau}\right]^{1/(1-\tau)}}{\left[\nu + (1-\nu)(P_{F,t}/P_{H,t})^{1-\theta}\right]^{1/(1-\theta)}} \cdot \frac{1}{Z_{x,t}}$$
(26)

As in Basu and Thoenissen (2011), the terms of trade  $P_{F,t}/P_{H,t}$  can create a wedge between the relative price of investment  $(P_{x,t}/P_t)$  and the IST shock,  $Z_{x,t}$ . A change in  $Z_{x,t}$  has a direct effect on the relative price of investment goods and an indirect effect working through the terms of trade. These two-pronged effects of IST on the relative price of investment makes it a major driver in business cycle fluctuation which we will see later.

## 4.4 Intermediate goods producing firms

As in Kollmann (2002), intermediate goods firms produce tradeable intermediate goods which can be used for consumption and investment by both home and foreign countries. These firms rent capital and hire labour from home households using the following constant returns to scale production function:

$$Y_t(i) = A_t K_t^{\alpha}(i) L_t^{1-\alpha}(i)$$

$$\tag{27}$$

where  $A_t$  is total factor productivity (TFP). Cost minimisation means:

$$\frac{K_t(i)}{L_t(i)} = (1-\alpha)\alpha^{-1} \frac{W_t}{R_{k,t}}$$

$$\tag{28}$$

where  $W_t$  and  $R_{k,t}$  are the nominal wage and nominal rental price plus depreciation cost. It is straightforward to verify that the nominal marginal cost is:

$$MC_t = \frac{1}{A_t} R^{\alpha}_{k,t} W^{1-\alpha}_t \alpha^{-\alpha} (1-\alpha)^{\alpha-1}$$
(29)

Because intermediate goods firms produce differentiated goods, they are monopolistic price setters. Price setting is staggered as in Calvo (1983). These firms set  $P_{H,t}$  after receiving a price signal that  $\gamma_p$  fraction of firms will keep the price unchanged in the next period. They also take the demand functions of their intermediate goods as given. Under the assumption that there is only cross border difference in demand elasticities for intermediate consumption and investment goods, there are only two relevant sequences of demand functions to be considered. These are as follows:

Domestic Demand: 
$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t}$$
  
Export Demand:  $Y_{H,t}^*(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon^*} Y_{H,t}^*$ 

where

$$Y_{H,t}(i) = C_{H,t}(i) + X_{H,t}(i)$$
(30)

$$Y_{H,t}^*(i) = C_{H,t}^*(i) + X_{H,t}^*(i)$$
(31)

and

$$Y_{H,t} = C_{H,t} + X_{H,t} (32)$$

$$Y_{H,t}^* = C_{H,t}^* + X_{H,t}^*$$
(33)

The profit of the home intermediate goods firms is given by:

$$\Omega_t^d(P_{H,t}, P_{H,t}^*) = \left[ (P_{H,t}(i)Y_{H,t}(i) + P_{H,t}^*(i)Y_{H,t}^*(i) - \Psi(Y_{H,t}(i) + Y_{H,t}^*(i))) \right]$$
(34)

where  $\Psi(.)$  is the nominal cost of production.

The dynamics of prices across two segmented markets (subject to identical nominal friction) can be written as:

$$P_{H,t} = \left[\gamma_p \left(P_{H,t-1}\Pi\right)^{1-\varepsilon} + \left(1-\gamma_p\right) \left(\widetilde{P}_{H,t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$
(35)

$$P_{H,t}^* = \left[\gamma_p \left(P_{H,t-1}^*\Pi\right)^{1-\varepsilon^*} + \left(1-\gamma_p\right) \left(\widetilde{P}_{H,t}^*\right)^{1-\varepsilon^*}\right]^{\frac{1}{1-\varepsilon^*}}$$
(36)

where "" stands for the optimal price.

## 4.4.1 Price setting equations

Home price is determined by the following price setting problem :

$$\widetilde{P}_{H,t} = \arg\max_{\varrho_t} \sum_{k=0}^{\infty} \gamma_p^k E_t \left[ D_{t,t+k} \left\{ \Pi^k \varrho_t \left( \frac{\varrho_t}{P_{H,t+k}} \right)^{-\varepsilon} Y_{H,t+k} - \Psi(Y_{t+k}) \right\} \right]$$
(37)

Since prices are non-stationary, we deflate the domestic price by CPI deflator. By doing this, we get the following linearised price setting equation around a constant steady state inflation,  $\Pi$ .

$$=>\frac{\widetilde{P}_{H,t}}{P_t} = \left(1 - \overline{\beta}\right)\frac{\varepsilon}{\varepsilon - 1}MC_t + \overline{\beta}E_t\frac{\widetilde{P}_{H,t+1}}{P_{t+1}}$$
(38)

where

$$\overline{\beta} = \gamma_p \beta \Pi^{\varepsilon}$$

As in Kollmann (2002), home country sets the export price in foreign currency and solves the following price setting problem analogous to the domestic prices.

$$\widetilde{P}_{H,t}^{*} = \arg\max_{\varkappa_{t}} \sum_{k=0}^{\infty} \gamma_{p}^{k} E_{t} \left[ D_{t,t+k} \left\{ \xi_{t+k} \Pi^{*^{k}} \varkappa_{t} \left( \frac{\varkappa_{t} \Pi^{*^{k}}}{P_{H,t+k}} \right)^{-\varepsilon^{*}} Y_{H,t+k}^{*} - \Psi(Y_{t+k}) \right\} \right]$$
(39)

which can be written in a loglinearised form around a constant foreign inflation rate,  $\Pi^*$  analogous to (38) as follows

$$=>\frac{\widetilde{P}_{H,t}^{*}\xi_{t}}{P_{t}}=\frac{(1-\beta^{*})\varepsilon^{*}}{\varepsilon^{*}-1}.MC_{t}+\beta^{*}E_{t}\frac{\widetilde{P}_{H,t+1}^{*}\xi_{t+1}}{P_{t+1}}$$
(40)

where

 $\beta^* = \beta \gamma^p {\Pi^*}^{-\varepsilon^*}$ 

## 4.5 Fiscal policy

The home government consumes  $G_t$  of final consumption goods and finances this by lump sum taxes  $T_t$  and borrowing. The government issues bonds which are domestically held. In other words, the government budget constraint is:

$$P_t G_t - T_t = \frac{B_{H,t+1}}{1+i_t} - B_{H,t} \tag{41}$$

The government spending  $G_t$  is the single fiscal policy tool which is formulated as the following process:

$$\ln G_t - \ln \overline{G} = \rho_G(\ln G_{t-1} - \ln \overline{G}) + \rho_{gy} \ln Y_{H,t-1} + \xi_t^g$$
(42)

where  $\overline{G}$  is the steady state government spending,  $\rho_G \in (0, 1)$  is the persistence of fiscal spending,  $\xi_t^g$  is the fiscal policy white noise shock. We assume that government spending is countercyclical which explains the second term on the right hand side where  $\rho_{gy}$  is negative.

## 4.6 Monetary policy

The CB sets an interest rate rule  $(i_t)$  and it auctions off  $\{B_{H,t}\}$  sequence of bonds at that mandated interest rate to finance its spending stream  $\{G_t\}$ . Any shortfall is financed by lump sum taxation,  $\{T_t\}$  which obeys the government budget constraint.

The interest rate sequence follows a standard Taylor rule in the short run and is specified as follows.

$$i_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_y \hat{y}_{H,t} + \xi_t^m$$
(43)

where ' $\hat{}$ ' represents the proportional deviation from the steady state. The monetray authrity responds by raising the interest rate  $(i_t)$  if it ancipates a higher inflation rate  $(\pi_{t+1})$  or experiences a higher output gap  $(y_{H,t})$ .  $\xi_t^m$  is the monetary policy white noise shock. We assume that monetary authorities in both home and foreign countries target respective inflation rates which are achievable in the long run.

## 4.7 Market equilibrium

The solution of our model satisfies the following market equilibrium conditions which must hold for the home and foreign country:

1. Home-produced intermediate goods market clears:

$$Y_t = Y_{H,t} + Y_{H,t}^*$$
(44)

2. Foreign-produced intermediate goods market clears:

$$Y_t^* = Y_{F,t} + Y_{F,t}^* \tag{45}$$

3. Bond Market clears:

$$\frac{\xi_t B_{F,t}}{P_t (1+i_t^*) \Theta\left(\frac{\xi_t B_{F,t}}{P_t}\right)} - \frac{\xi_t B_{F,t-1}}{P_t} = \frac{\xi_t P_{H,t}^*}{P_t} Y_{H,t}^* - \frac{P_{F,t}}{P_t} Y_{F,t}$$
(46)

and

$$\sum_{j} B_{H,t}^j = B_{H,t} \tag{47}$$

where  $B_{H,t}$  follows the government budget constraint.

Note that the first equality in (46) shows the current account balance. The right hand side is home country's net export.

## 4.8 National income accounting

It is straightforward to verify that the Walras law holds for the aggregate economy. Aggregation of the flow budget constraints of all home households yields:

$$P_{t}\sum_{j}C_{t}^{j} + P_{x,t}\sum_{j}X_{t}^{j} + \frac{\sum_{j}B_{H,t+1}^{j}}{(1+i_{t})} + \frac{\xi_{t}\sum_{j}B_{F,t}^{j}}{(1+i_{t}^{*})\Theta\left(\frac{\xi_{t}B_{F,t}}{P_{t}}\right)}$$
$$= \sum_{j}B_{H,t}^{j} + \xi_{t}\sum_{j}B_{F,t-1}^{j} + W_{t}\sum_{j}L_{t}^{j} + \sum_{j}R_{k,t}K_{t}^{j} + \sum_{j}\Omega_{t}^{d,j} - \sum_{j}T_{t}^{j}$$

Then, substituting the bond market clearing condition in (46) we get:

$$P_t C_t + P_{x,t} X_t + \frac{B_{H,t+1}}{(1+i_t)} - B_{H,t} + P_{H,t}^* \xi_t Y_{H,t}^* - P_{F,t} Y_{F,t} = W_t L_t + R_{k,t} K_t + \Omega_t^d - T_t$$
(48)

However the aggregate profit is given by (using the market clearing condition (44)):

$$\Omega_t^d = P_{H,t}Y_t - W_t L_t - R_{k,t}K_t \tag{49}$$

which after plugging into (48) yields

$$P_t C_t + P_{x,t} X_t + \frac{B_{H,t+1}}{(1+i_t)} - B_{H,t} + P_{H,t}^* \xi_t Y_{H,t}^* - P_{F,t} (C_{F,t} + X_{F,t}) = P_{H,t} Y_t - T_t$$
(50)

Finally, we substitute the government budget constraint to get rid of the tax term and obtain:

$$P_t C_t + P_{x,t} X_t + P_t G_t + P_{H,t}^* \xi_t Y_{H,t}^* - P_{F,t} Y_{F,t} = P_{H,t} Y_t$$
(51)

Note that  $(P_{H,t}^* \xi_t Y_{H,t}^* - P_{F,t} Y_{F,t})$  is home country's net export.

Hence, the national income identity is verified.

## 4.9 Modified uncovered interest parity condition

From (9) and (10),

$$\frac{1+i_t}{1+i_t^*} = E_t \left(\frac{\xi_{t+1}}{\xi_t}\right) \Theta \left(\frac{\xi_t B_{F,t}}{P_t}\right)$$
(52)

The bond holding cost function  $\Theta(.)$  drives wedge between home and foreign bond returns.

## 4.10 Foreign country

As in Benigno (2009), the foreign country's problem is symmetric. We assume that the foreign country uses the same consumption basket as the home country which makes the purchasing parity to hold in the long run, i.e.  $P_t = \xi_t P_t^*$ ,  $P_{H,t} = \xi_t P_{H,t}^*$  and  $P_{F,t} = \xi_t P_{F,t}^*$  where  $\xi_t$  is the nominal exchange rate. Foreign country has an additional Euler equation involving foreign bond which is the only tradeable financial asset and is given by:

$$V_{C,t} = \beta (1+i_t^*) E_t \left[ V_{C,t+1} \cdot \left( \frac{P_t^*}{P_{t+1}^*} \right) \right]$$
(53)

where  $V_{C,t}$  is the marginal utility of foreign consumption at date t.

# 4.11 Short run dynamics

The short run dynamics can be summarised by the following 29 equations:

1. Domestic Price Setting Equation

$$\frac{\widetilde{P}_{H,t}}{P_t} = (1 - \overline{\beta}) \frac{\varepsilon}{\varepsilon - 1} M C_t + \overline{\beta} E_t \frac{\widetilde{P}_{H,t+1}}{P_{t+1}}$$
(54)

2. Export Price Setting Equation

$$\frac{\tilde{P}_{H,t}^{*}\xi_{t}}{P_{t}} = \frac{(1-\beta^{*})\varepsilon^{*}}{\varepsilon^{*}-1} MC_{t} + \beta^{*}E_{t}\frac{\tilde{P}_{H,t+1}^{*}\xi_{t+1}}{P_{t+1}}$$
(55)

3. Dynamics of  $P_{H,t}/P_t$ 

$$\frac{P_{H,t}}{P_t} = \left[\gamma_p \left(\Pi/\Pi_t\right)^{1-\theta} \left(\frac{P_{Ht-1}}{P_{t-1}}\right)^{1-\theta} + (1-\gamma_p) \left(\frac{\widetilde{P}_{H,t}}{P_t}\right)^{1-\theta}\right]^{1/(1-\theta)}$$
(56)

4. Dynamics of  $\frac{P_{H,t}^*\xi_t}{P_t}$ 

$$\frac{P_{H,t}^{*}\xi_{t}}{P_{t}} = \left[\gamma_{p}\left(\frac{\xi_{t-1}P_{H,t-1}^{*}\Pi^{*}}{P_{t-1}}\right)^{1-\theta}\left(\frac{\xi_{t}/\xi_{t-1}}{P_{t}/P_{t-1}}\right)^{1-\theta} + (1-\gamma_{p})\left(\frac{\widetilde{P_{H,t}^{*}\xi_{t}}}{P_{t}}\right)^{1-\theta}\right]^{1/(1-\theta)}$$
(57)

5.  $P_{H,t}/P_t$  equation: (PPI/CPI aggregator)

$$\frac{P_{H,t}}{P_t} = \left[\nu + (1-\nu)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\theta}\right]^{1/(\theta-1)}$$
(58)

6.  $P_{x,t}/P_t$  equation (from CPI aggregator)

$$\frac{P_{x,t}}{P_t} = \frac{\left[\varphi + (1-\varphi)(P_{F,t}/P_{H,t})^{1-\tau}\right]^{1/(1-\tau)}(1/Z_t^x)}{\left[\nu + (1-\nu)(P_{F,t}/P_{H,t})^{1-\theta}\right]^{1/(1-\theta)}}$$
(59)

7.  $P_{F,t}/P_t$  equation

$$P_{F,t}/P_t = (P_{F,t}/P_{H,t})(P_{H,t}/P_t)$$
(60)

8.  $P_{x,t}/P_{H,t}$  equation

$$P_{x,t}/P_{H,t} = (P_{x,t}/P_t)/(P_{H,t}/P_t)$$
(61)

9. Tobin's q equation

$$q_t \left[ (1 - s(X_t/X_{t-1})) - s'(X_t/X_{t-1})(X_t/X_{t-1}) \right] + E_t q_{t+1} s'(X_{t+1}/X_t)(X_{t+1}/X_t)^2 m_{t+1} = P_{x,t}/P_t$$
(62)

10. Dynamic Efficiency Condition

$$q_t = m_{t+1} \left[ q_{t+1}(1-\delta) + \frac{R_{k,t+1}}{P_{t+1}} \right]$$
(63)

11. SDF

$$m_{t+1} = \frac{\beta (C_{t+1} - \gamma_c C_t)^{-\sigma_c}}{(C_t - \gamma_c C_{t-1})^{-\sigma_c}}$$
(64)

12. Cost Minimisation

$$\frac{W_t}{P_t} = \frac{1 - \alpha}{\alpha} \left(\frac{R_{k,t}}{P_t}\right) \cdot \left(\frac{K_t}{L_t}\right) \tag{65}$$

13. Capital Stock Dynamics

$$K_{t+1} = (1 - \delta)K_t + (1 - S(X_t/X_{t-1}))X_t$$
(66)

14. Static Efficiency Condition

$$L_t^{\sigma_L} = (C_t - \gamma_c C_{t-1})^{-\sigma_c} (W_t / P_t)$$
(67)

15. Real MC equation

$$MC_t = \frac{1}{A_t} (R_{k,t}/P_t)^{\alpha} (W_t/P_t)^{1-\alpha} \alpha^{-\alpha} (1-\alpha)^{\alpha-1}$$
(68)

16. Domestic Demand Function

$$Y_{H,t} = v \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t + \varphi \left(\frac{P_{H,t}}{P_{x,t}}\right)^{-\tau} X_t$$
(69)

17. Import Demand Function:

$$Y_{F,t} = C_{F,t} + X_{F,t}$$

$$\Rightarrow Y_{F,t} = \left[ (1 - \nu)(P_{F,t}/P_t)^{-\theta} C_t + (1 - \varphi)(P_{F,t}/P_{x,t})^{-\tau} X_t \right]$$
(70)

18. National Income Identity,

$$P_t C_t + P_{x,t} X_t + P_t G_t + \xi_t P_{H,t}^* Y_{H,t}^* - P_{F,t} Y_{F,t} = P_{H,t} A_t F(K_t, L_t)$$
  
$$\Rightarrow (P_t/P_{H,t}) C_t + (P_{x,t}/P_{H,t}) X_t + (P_t/P_{H,t}) G_t + (\xi_t P_{H,t}^*/P_{H,t}) Y_{H,t}^* - (P_{F,t}/P_{H,t}) Y_{F,t} = A_t F(K_t, L_t)$$
  
(71)

19. Home market clearing condition

$$A_t F(K_t, L_t) = Y_{H,t} + Y_{H,t}^*$$
(72)

20. Consumer Euler Eqn

$$\frac{1}{1+i_t} \cdot \frac{\beta (C_{t+1} - \gamma_c C_t)^{-\sigma_c}}{(C_t - \gamma_c C_{t-1})^{-\sigma_c}} \cdot (1 + \pi_{t+1}) = 1$$
(73)

21. Home Taylor rule for home interest rate

$$\hat{i}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_y \hat{y}_{H,t} + \xi_t^m \tag{74}$$

where  $\phi_{\pi} > 0, \phi_{y} > 0$ . The first two responsiveness of the Central Bank interest rate rule is

standard. The last term reflects the responsiveness of the policy authority to domestic exchange rate depreciation.

22. Foreign asset dynamics (comes from (46))

$$\frac{b_{F,t}}{(1+i_t^*)\Theta(b_{F,t})} - \frac{(\xi_t/\xi_{t-1})b_{F,t-1}}{(1+\pi_t)} = \frac{\xi_t P_{H,t}^* Y_{H,t}^*}{P_t} - \frac{P_{F,t}}{P_t} Y_{F,t}$$
(75)

where  $b_{F,t} = \frac{\xi_t B_{F,t}}{P_t}$ 

23. UIP pinning down  $\bigtriangleup \xi_{t+1}$  Eqn (52)

$$\frac{1+i_t}{1+i_t^*} = E_t\left(\frac{\xi_{t+1}}{\xi_t}\right) \cdot \Theta\left(\frac{\xi_t B_{F,t}}{P_t}\right)$$
(76)

24. Next Export  $(NX_t)$  Function

$$NX_t = \frac{\xi_t P_{H,t}^*}{P_t} Y_{H,t}^* - \frac{P_{F,t}}{P_t} Y_{F,t}$$
(77)

25. Forcing process for TFP

$$\ln A_t - \ln A = \rho_a \{ \ln A_{t-1} - \ln A \} + \xi_t^a \tag{78}$$

26. Forcing process for IST

$$\ln Z_{x,t} - \ln Z_x = \rho_z \{\ln Z_{x,t-1} - \ln Z_x\} + \xi_t^z \tag{79}$$

27. Forcing process for fiscal spending shock

$$\ln G_t - \ln \overline{G} = \rho_g \{ \ln G_{t-1} - \ln \overline{G} \} + \rho_{gy} \ln Y_{H,t-1} + \xi_t^g$$
(80)

28. Forcing process for foreign interest rate

$$i_t^* - i^* = \rho_i (i_{t-1}^* - i^*) + \xi_t^i \tag{81}$$

29. Forcing process for export demand

$$Y_{H,t}^* - \bar{Y_H^*} = \rho_{\exp}(Y_{H,t-1}^* - \bar{Y_H^*}) + \xi_{\exp,t}$$
(82)

## 4.12 Long run model

Hereafter variables except nominal prices are without subscript. Note that all nominal prices are non-stationary which means ratio of two such prices is stationary.

Note that the steady state system has a partial recursiveness property as far as relative prices are concerned if we involve long run LOOP.<sup>4</sup>

$$\frac{P_{F,t}}{P_{H,t}} = 1 \tag{83}$$

Next, using (24) solve the relative price of investment as follows

$$\frac{P_{x,t}}{P_t} = 1 \tag{84}$$

which also implies

$$\frac{P_{H,t}}{P_t} = 1 \tag{85}$$

Using (84) and (62)

q = 1

which after substitution in (63) pins down the steady state real rental price of capital.

$$r_k = \frac{R_{k,t}}{P_t} = \left[\frac{1-\beta}{\beta} + \delta\right] \tag{86}$$

From (54),

$$MC = \frac{\varepsilon - 1}{\varepsilon}$$

<sup>&</sup>lt;sup>4</sup>As in Benigno (2009), to invoke the LOOP we need to assume that both home and foreign countries consume the same commodity basket. which means  $\nu = 1 - \nu^*$ . To see it write respective prices as follows:  $P_t = [vP_{H,t}^{1-\theta} + (1-v)P_{F,t}^{1-\theta}]^{1/(1-\theta)}$  and  $P_t^* = (1/\xi_t)[v^*P_{F,t}^{1-\theta^*} + (1-v^*)P_{H,t}^{1-\theta^*}]^{1/(1-\theta^*)}$  where  $\xi_t$  is the nominal exchange rate. If  $\nu = 1 - \nu^*$ , then  $P_{H,t}/P_{F,t} = 1 = > P_t = \xi_t P_t^*$ .

and likewise from (55),

$$\frac{P_{H,t}^*\xi_t}{P_t} = \frac{\varepsilon^*}{\varepsilon^* - 1} \frac{\varepsilon - 1}{\varepsilon}$$
(87)

Next note from (68) that

$$\frac{\varepsilon - 1}{\varepsilon} = \frac{1}{A} (R_{k,t}/P_t)^{\alpha} (W_t/P_t)^{1 - \alpha} \alpha^{-\alpha} (1 - \alpha)^{\alpha - 1}$$
(88)

Given  $r_k$  from (86), one can solve w,

$$w = W_t / P_t = \left[\frac{\frac{\varepsilon - 1}{\varepsilon} A \alpha^{\alpha} (1 - \alpha)^{1 - \alpha}}{r_k^{\alpha}}\right]^{1/(1 - \alpha)}$$
(89)

Once w and  $r_k$  are determined, using the cost minimization condition (65), the optimal K : L ratio can be determined as follows:

$$L = [(\alpha/(1-\alpha))(w/r_k)]^{-1} K$$
(90)

$$= BK$$
 (91)

Next, we use (66) to determine the steady state investment,

$$X = \delta K \tag{92}$$

Further, using (67),

$$L^{\sigma_L}C^{\sigma_c} = w(1-\gamma_c)^{-\sigma_c}$$
if  $\sigma_L = 0$  and  $\sigma_c = 1 \Longrightarrow C = \frac{w}{(1-\gamma_c)}$ 

$$(93)$$

Next, note from GDP definition that

$$Y_H + Y_H^* = AK^{\alpha}L^{1-\alpha}$$

Plugging (90) into above

$$Y_H + Y_H^* = A^* K$$
  
where  $A^* = AB^{1-\alpha}$ 

Given (85) and (92), note from (69) that

$$Y_H = vC + \varphi \delta K$$

=>

$$vC + \varphi \delta K = A^*K - Y_H^*$$

But from static efficiency condition, we get:  $C = \frac{w}{(1-\gamma_c)}$  which after plugging into above solves

$$K = \frac{v \frac{w}{(1-\gamma_c)} + Y_H^*}{[A^* - \varphi \delta]} \tag{94}$$

Plugging K into (90) we get L.

Next, we need to solve G. From the national income identity (71), we obtain:

$$C + \delta K + G + (\xi_t P_{H,t}^* / P_{H,t}) Y_H^* - Y_F = A K^{\alpha} L^{1-\alpha}$$
(95)

Note that  $\xi_t P_{H,t}^* / P_{H,t} = \frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{1 - \varepsilon}{\varepsilon}$ . Using (87), we rewrite (95) as:

$$C + \delta K + G + \left(\frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{\varepsilon - 1}{\varepsilon}\right) Y_H^* - Y_F = A^* K$$
(96)

but using the import demand function (70),  $Y_F = (1 - \nu)C + (1 - \varphi)X$ , which after substituting in (96),

$$C + \delta K + G + \left(\frac{\varepsilon^*}{\varepsilon^* - 1}, \frac{\varepsilon - 1}{\varepsilon}\right) Y_H^* - \left((1 - \nu)C + (1 - \varphi)X\right) = A^*K$$

Since  $X = \delta K$ 

$$C + \delta K + G + \left(\frac{\varepsilon^*}{\varepsilon^* - 1}, \frac{\varepsilon - 1}{\varepsilon}\right) Y_H^* - (1 - \nu)C - (1 - \varphi)\delta K = A^*K$$

=>

$$\nu C + \varphi \delta K + G + \left(\frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{\varepsilon - 1}{\varepsilon}\right) Y_H^* = A^* K$$

Substituting (93),

$$\frac{\nu w}{(1-\gamma_c)} + \varphi \delta K + G + \left(\frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{\varepsilon - 1}{\varepsilon}\right) Y_H^* = A^* K$$

As we know K from (94), we can solve G as follows:

$$G = (A^* - \varphi \delta)K - \frac{\nu w}{(1 - \gamma_c)} - \left(\frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{\varepsilon - 1}{\varepsilon}\right)Y_H^*$$

Note that G is actually net transfer which can be positive or negative.

Next, we solve the steady state foreign bond holding. Using (75) and evaluating the same at the steady state (using the fact that the steady state bond holding spread  $\Theta(b_f) = 1$ ), to get:

$$b_f\left[\frac{1}{1+i^*} - \frac{\Delta\xi}{1+\pi}\right] = \frac{\varepsilon^*}{\varepsilon^* - 1} . MC. Y_H^* - Y_F$$

where  $\Delta \xi = \xi_t / \xi_{t-1}$  and

$$Y_F = C_F + X_F$$
  
=  $(1 - v)C + (1 - \varphi).\delta K$  (97)

Using steady state UIP,

$$b_f\left[\frac{\Delta\xi}{1+i} - \frac{\Delta\xi}{1+\pi}\right] = \frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot Y_H^* - Y_F$$

Using the Fisher relation  $1 + i = (1 + \pi)/\beta$ ,

$$b_f \frac{(\beta - 1)\Delta\xi}{1 + \pi} = \frac{\varepsilon^*}{\varepsilon^* - 1} . MC. Y_H^* - Y_F$$

Using the purchasing power parity condition,

$$b_f \frac{(\beta - 1)}{1 + \pi^*} = \frac{\varepsilon^*}{\varepsilon^* - 1} . MC. Y_H^* - Y_F$$

$$b_f = \frac{1+\pi^*}{(1-\beta)} \left[ Y_F - \frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot Y_H^* \right]$$
$$= \frac{1+\pi^*}{(1-\beta)} \left[ (1-v)C + (1-\varphi) \cdot \delta K - \frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot Y_H^* \right]$$

Now, plugging the solutions for C and K into above

$$b_f = \frac{1+\pi^*}{(1-\beta)} \left[ (1-v) \left\{ \frac{w}{(1-\gamma_c)} \right\} + (1-\varphi) \cdot \delta \left\{ \frac{v \frac{w}{(1-\gamma_c)} + Y_H^*}{[A^* - \varphi \delta]} \right\} - \frac{\varepsilon^*}{\varepsilon^* - 1} \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot Y_H^* \right]$$

Finally, it is straightforward to check from the forcing processes and the inflation targeting that

$$1 + i = \frac{1 + \pi^{target}}{\beta}$$
$$\pi^* = \pi^{*target}$$
$$i_t^* = i^*$$

$$\Delta \xi_t = \pi^{target} - \pi^{*target}$$

$$A_t = A$$

$$Z_t^x = Z^x$$

#### 4.13 Solution strategy

As in Kollmann (2002), we specialise to a utility function  $[\ln(C_t - \gamma_c C_{t-1}) - L_t]$ . We loglinearise the non-linear optimal conditions and the resource constraints around the steady state of the respective variables as described in the long run model. This produces a system of equations which is listed below.

ow. 1. Home price setting:  $\widehat{\frac{\widehat{P_{H,t}}}{P_t}} = (1 - \overline{\beta}) \frac{\varepsilon}{\varepsilon - 1} \overline{mc} \widehat{mc}_t + \overline{\beta} E_t \left( \widehat{\frac{P_{H,t+1}}{P_{t+1}}} \right)$ 2. Export price setting:  $\widehat{\frac{\xi_t P_{H,t}^*}{P_t}} = (1 - \beta^*) \frac{\varepsilon^*}{\varepsilon^* - 1} \overline{mc} \widehat{mc}_t + \beta^* E_t \left( \underbrace{\frac{\xi_{t+1} P_{H,t+1}^*}{P_{t+1}}}_{(-)} \right)$ 

2. Export price setting: 
$$\frac{\xi_t P_{H,t}^*}{P_t} = (1 - \beta^*) \frac{\varepsilon^*}{\varepsilon^* - 1} \overline{mc} \widehat{mc}_t + \beta^* E_t \left( \frac{\xi_{t+1} P_{H,t}^*}{P_{t+1}} \right)$$

- 3. Home Price Dynamics:  $\frac{\widehat{P_{H,t}}}{P_t} = \gamma_p \left[ \left( \frac{\widehat{P_{H,t-1}}}{P_{H,t}} \right) \hat{\pi}_t \right] + (1 \gamma_p) \left( \frac{\widehat{P_{H,t}}}{P_{H,t}} \right)$ 4. Export Price Dynamics:
- 4. Export Price Dynamics:

$$\frac{\widehat{\xi_t P_{H,t}^*}}{P_t} = \Theta_1 \left\{ \gamma_p \left( \frac{\Pi^*}{\Pi} \right)^{1 - \varepsilon^*} \overline{\Delta \xi}^{1 - \varepsilon^*} \left[ \left( \frac{\widehat{\xi_{t-1} P_{H,t-1}^*}}{P_{t-1}} \right) + \widehat{\Delta \xi} - \widehat{\pi}_t \right] + (1 - \gamma_p) \left( \frac{\widehat{\xi_t P_{H,t}^*}}{P_t} \right) \right\}$$

where

$$\begin{aligned} \Theta_{1} &= \frac{1}{\gamma_{p}(\Pi^{*}/\Pi)^{1-\varepsilon} + (1-\gamma_{p})} \\ \text{Price System} \\ 5. \quad & \frac{\widehat{P_{H,t}}}{P_{t}} = (\nu-1)\frac{\widehat{P_{H,t}}}{P_{F,t}} \\ 6. \quad & \frac{\widehat{P_{X,t}}}{P_{t}} = (\nu-\varphi)\frac{\widehat{P_{F,t}}}{P_{X,t}} \\ 7. \quad & \frac{\widehat{P_{F,t}}}{P_{t}} = \frac{\widehat{P_{F,t}}}{P_{t}} + \frac{\widehat{P_{H,t}}}{P_{t}} \\ 8. \quad & \frac{\widehat{P_{X,t}}}{P_{H,t}} = \frac{\widehat{P_{X,t}}}{P_{t}} - \frac{\widehat{P_{H,t}}}{P_{t}} \\ 9. \text{ Tobin's q: } \hat{q}_{t} = (1+\beta)s''(1)\hat{x}_{t} - s''(1)\hat{x}_{t-1} - \beta E_{t}\left(s''(1)\hat{x}_{t+1}\right) + \frac{\widehat{P_{X,t}}}{P_{t}} \\ 10. \text{ Dynamic Efficiency Condition: } \hat{q}_{t} = \hat{m}_{t+1} + \left((1-\delta)\hat{q}_{t+1} + \widehat{r}_{k}\hat{r}_{k,t+1}\right)/(\widehat{r}_{k} + 1-\delta) \\ 11. \text{ Stochastic Discount Factor: } \hat{m}_{t+1} = (1+\gamma_{c}) \cdot \frac{\sigma_{c}}{1-\gamma_{c}} \cdot \hat{c}_{t} - \frac{\gamma_{c}\sigma_{c}}{1-\gamma_{c}} \cdot \hat{c}_{t-1} \\ 12. \text{ Cost Minimization: } \hat{w}_{t} = \hat{r}_{k,t} + \hat{k}_{t} - \hat{l}_{t} \\ 13. \text{ Capital Stock Dynamics: } \hat{k}_{t+1} = (1-\delta)\overline{K}\hat{k}_{t} + \overline{X}\hat{x}_{t} \\ 14. \text{ Static Efficiency Condition: } \hat{c}_{t} = \gamma_{c}\overline{c}\hat{c}_{t-1} + \left(\frac{1-\gamma_{c}}{\sigma_{c}\overline{c}}\right)\hat{w}_{t} \\ 15. \text{ Real MC: } \hat{m}_{c_{t}} = \alpha\hat{r}_{k,t} + (1-\alpha)\hat{w}_{t} - \hat{A}_{t} \end{aligned}$$

16. Domestic Demand:  $\hat{y}_{H,t} = \frac{1}{\nu \bar{C} + \psi \bar{X}} \left[ \nu \bar{C} \hat{c}_t - \nu \theta \bar{C} \left( \frac{\widehat{P}_{H,t}}{P_t} \right) + \psi \bar{X} \hat{x}_t - \psi \tau \bar{X} \left( \frac{\widehat{P}_{H,t}}{P_{X,t}} \right) \right]$ 17. Import Demand:

$$\hat{y}_{F,t} = \left[ -(1-\nu)\theta\bar{C}\left(\frac{\widehat{P_{H,t}}}{P_t}\right) - \tau(1-\psi)\bar{X}\left(\frac{\widehat{P_{F,t}}}{P_{X,t}}\right) + (1-\nu)\bar{C}\hat{c}_t + (1-\psi)\bar{X}\hat{x}_t \right]$$

where  $\Theta_2 = \frac{1}{(1-\nu)\bar{C}+(1-\psi)\bar{X}}$ 18 National Income Identit

18. National Income Identity:

$$\frac{\bar{C}\hat{c}_t + \bar{X}\left(\frac{\widehat{P_{X,t}}}{P_t}\right) + \bar{X}\hat{x}_t + \bar{G}\hat{g}_t + \left(\frac{\varepsilon^*}{\varepsilon^* - 1}\frac{\varepsilon - 1}{\varepsilon}\right)\bar{Y}_H\left[\left(\frac{\widehat{Y}_{H,t}}{P_t}\right) + \hat{y}_{H,t}^*\right] - \bar{Y}_F\left[\left(\frac{\widehat{P}_{F,t}}{P_t}\right) + \hat{y}_{F,t}\right]}{\bar{C} + \bar{X} + \bar{G} + \left(\frac{\varepsilon^*}{\varepsilon^* - 1}\frac{\varepsilon - 1}{\varepsilon}\right)\bar{Y}_H^* - \bar{Y}_F}$$
$$= \left(\frac{\widehat{P}_{H,t}}{P_t}\right) + \hat{A}_t + \alpha\hat{k}_t + (1 - \alpha)\hat{l}_t$$

19. Home Market Clearing:  $\hat{A}_t + \alpha \hat{k}_t + (1 - \alpha)\hat{l}_t = \bar{Y}_H \hat{y}_{H,t} + \bar{Y}_H^* \hat{y}_{H,t}^*$ 

- 20. Consumer Euler Eqn:  $\frac{\pi^{target}}{1+\pi^{target}}\hat{\pi}_{t+1} \frac{\bar{\imath}}{1+\bar{\imath}}\hat{\imath}_t + E_t\left(\hat{m}_{t+1}\right) = 0$
- 21. Taylor Rule:  $i_t = \phi_{\pi} E_t \hat{\pi}_{t+1} + \phi_y \hat{y}_{H,t} + \xi_t^m$
- 22. Foreign Asset Dynamics

$$\begin{bmatrix} \overline{b}_F \\ \overline{1+\overline{\imath^*}} + \frac{\overline{b}_F^2 \Theta'(\overline{b}_F)}{(1+\overline{\imath^*})} \end{bmatrix} \hat{b}_{F,t} - \frac{\overline{b}_F \overline{\imath^*}}{(1+\overline{\imath^*})^2} \hat{\imath}_t^* - \frac{\overline{\Delta\xi} \overline{b}_F}{1+\overline{\Pi}} \hat{b}_{F,t-1} - \frac{\overline{b}_F \overline{\Delta\xi}}{1+\overline{\Pi}} \widehat{\Delta\xi}_t + \frac{\overline{\Delta\xi} \overline{b}_F \Pi}{(1+\overline{\Pi})^2} \hat{\pi}_t$$
$$= \frac{\varepsilon^*}{\varepsilon^* - 1} \overline{mc} \overline{Y}_H^* \left( \widehat{\underline{\xi}_t P_{H,t}^*} \right) + \frac{\varepsilon^*}{\varepsilon^* - 1} \overline{mc} \overline{Y}_H^* \hat{y}_{H,t}^* - \overline{Y}_F \hat{y}_{F,t} - \overline{Y}_F \left( \frac{\widehat{P}_{F,t}}{P_t} \right)$$
23. UIP:  $\frac{\overline{\imath}}{1+\overline{\imath}} \hat{\imath}_t - \frac{\overline{\imath^*}}{1+\overline{\imath}} \hat{\imath}^* = \frac{\overline{\Delta\xi}}{1+\overline{\Delta\xi}} \widehat{\Delta\xi}_t - \Theta'(\overline{b}_F) \overline{b}_F \hat{b}_F$ 

24. NX:  $NX_t = \overline{Y}_{H}^* \cdot \frac{\varepsilon^*}{\varepsilon^* - 1} \cdot (\frac{\xi_t P_{H,t}^*}{P_t} + Y_{H,t}) - \frac{\overline{P_F}}{P} \cdot \overline{Y_F} \cdot \overline{Y}_{F,t}$ 

Exogenous shock processes

25. TFP:  $\hat{A}_t = \rho_a \hat{A}_{t-1} + \hat{\xi}_t^a$ 26. IST:  $\hat{Z}_{x,t} = \rho_z \hat{Z}_{x,t-1} + \hat{\xi}_t^z$ 

- 27. Fiscal Spending:  $\hat{g}_t = \rho_g \hat{g}_{t-1} + \rho_{gy} \hat{y}_{H,t-1} + \hat{\xi}_t^g$
- 28. Foreign Interest Rate:  $\hat{\imath}_t^* = \rho_i \hat{\imath}_{t-1}^* + \hat{\xi}_t^i$
- 29. Export Demand:  $\hat{y}_{H,t}^* = \rho_{exp}\hat{y}_{H,t-1}^* + \xi_{exp,t}$

In the above system of equations, we have 29 equations with 29 unknowns which indicates that the model is solvable. We propose to solve the model in Dynare to obtain a rational expectation equilibrium solution.

# 5 Model Simulation

In this section, we report some preliminary results of the performance of our baseline model. The results reported here are instructive and presented to illustrate that the model is functional and can yield plausible results. There are 26 parameters whose values are fixed at levels based on past studies with an aim to target a few key second moments of the Indian economy. Tables 2 and 3 report the parameter values.<sup>5</sup> In principle, many of these parameters can be estimated given the availability of long macroeconomic time series. Such a task is beyond the scope of this study.<sup>6</sup>

 Table 2: Parameter Values

β	$\gamma_p$	ε	ε*	$\theta$	ν	$\phi$	$\tau$	$\gamma_c$	α	S"(1)	$\phi_{\pi}$	$\phi_y$	$\Theta'(\bar{b}_f)$
0.96	0.66	2	2	2	0.4	0.5	2	0.6	0.3	2	1.5	0.02	$0.001.\overline{c}$

Table 3: Second Momemnt Parameter Values of the Forcing Pro-	cesses
--	--------

$\rho_a$	$\rho_{\rm exp}$	$\rho_g$	$\rho_{gy}$	$\rho_z$	$\sigma_i^2$	$\sigma_{i^*}^2$	$\sigma_g^2$	$\sigma_a^2$	$\sigma_x^2$	$\sigma_{y_h^*}^2$
0.75	0.75	0.75	-0.1	0.75	1	1	1	1	1	1

Table 4 reports the key cross correlations of the model and the data.<sup>7</sup> The basic stylised facts of the Indian economy are well reflected by our calibrated baseline economy. Among these

<sup>&</sup>lt;sup>5</sup>Without any loss of generality, we fix all the standard deviation at unit levels to normalise the impulse responses. <sup>6</sup>In future, this section would be enriched by formal estimation and calibration.

<sup>&</sup>lt;sup>7</sup>Since the data are HP filtered, the same filter is also applied to the simulated series of the model with a penalty factor 100 appropriate for annual data.

stylised facts, most noteworthy are the procyclical patterns of consumption and investment and countercyclical property of inflation.

	Data	Model
$Corr(c, y_H)$	0.79	0.36
$Corr(x, y_H)$	0.38	0.56
$Corr(\pi, y_H)$	-0.49	-0.47
$\operatorname{Corr}(c,G)$	0.46	0.11
$\operatorname{Corr}(x,i)$	0.37	0.90
$Corr(\Delta\xi, i)$	0.45	0.99
$\boxed{\operatorname{Corr}(NX, p_f/p_h)}$	0.61	0.99

Table 4: Correlations (Actual and Model)

The countercyclical nature of inflation in the data is intriguing because it goes contrary to the conventional wisdom of output-inflation Phillips curve trade off. The reason for this anti Phillips curve nature of the Indian inflation data is not well explored in the literature. The intuition for this property can be understood by looking at individual impulse responses of inflation with respect to each of the primitive structural shocks in the model. These impulse responses are plotted in Figures 2 through 7 with the labels,  $yh = \hat{y}_{H,t}$ ,  $c = \hat{c}_t$ ,  $x = \hat{x}_t$ ,  $i = \hat{i}_t$ ,  $l = \hat{l}_t$ , pii =  $\pi_t$ , del\_xi =  $\widehat{\Delta\xi}_t$ , pfph =  $\frac{\widehat{P_{F,t}}}{P_{H,t}}$ ,  $nx = NX_t$ . Note that the model cross correlations between inflation and output is the summary of the impulse response time paths of GDP gap and inflation driven by the six shocks. The output and inflation are likely to be negatively correlated if the time paths of output and inflation following a shock negatively correlate. This happens for almost all the shocks except TFP and foreign demand shocks. One surprising result is that inflation rises in response to both TFP and IST shocks. This is because a higher output response to each of these two shocks drive up the marginal cost. Since inflation is positively related to the current and future sequences of marginal costs, inflation also rises. On the monetary policy front, a positive interest rate shock lowers the inflation through the monetary policy channel.

On the fiscal policy front, a positive fiscal spending shock financed by taxes causes an adverse wealth effect on the household which crowds out consumption and investment. The substitution effect on leisure tends to dominate which explains a slight fall in labour supply. The decline in inflation shows the reduction in marginal cost due to fall in GDP.

The procyclical responses of consumption and investment are primarily driven by the TFP and IST shocks which are reflected in the impulse responses. All these three key macroeconomic variables respond positively to TFP and IST shocks.

On the external front, a positive foreign demand shock raises net export demand and makes home currency appreciate in value. The terms of trade  $P_F/P_H$  appreciates because a rise in foreign demand encourages home intermediate goods producers to supply more. This puts upward pressure on the marginal cost and thus raises relative price of home produced intermediate goods,  $P_H/P$ . Since the terms of trade is positively related to  $P_H/P$ , it rises.



Figure 2: Effect of a TFP Shock



Figure 3: Effect of an IST Shock



Figure 4: Effect of a Monetary Policy Shock



Figure 5: Effect of a Fiscal Policy Shock



Figure 6: Effect of a Foreign Demand Shock



Figure 7: Effect of a Foreign Interest Rate Shock

Table 5 compares the model impact effects with their data counterparts. We take the logdifferenced stationary series of output, consumption, investment, employment, inflation, and depreciation of the nominal exchange rate. We also have a set of variables which can represent the exogenous variables like TFP, IST, fiscal spending, foreign economy output, domestic and foreign monetary policy shock. Running an unrestricted Vector Autoregression, we obtain the empirical impulse responses of output, consumption, investment, employment, inflation, and depreciation of the nominal exchange rate with respect to one standard error innovations of the exogenous variables. We consider the impact effects of the shocks obtained from the empirical impulse response functions and compare them with the theoretical one.<sup>8</sup> The results, found from the data and model comparison based on the impact effects, are rather mixed. As far as TFP shock is concerned, the model performs remarkably well.

Table 6 reports the volatility property of the model and compares these with the data. The volatility is measured by the standard deviation. Generally the model underpredicts the volatility

<sup>&</sup>lt;sup>8</sup>We consider the annual data for the sample period of 1971 to 2010.

		A		$Z_x$		G		i		$Y_H^*$		i*	
	Data	Model	Data	Model									
$Y_H$	+	+	+	0	+	-	0	0	0	-	0	+	
C	+	+	0	+	0	-	0	-	-	-	+	+	
X	+	+	0	+	+	-	+	-	+	-	0	+	
	-	-	0	-	-	-	+	0	+	+	+	+	
$\pi$	-	-	0	+	-	-	0	-	+	-	+	+	
$\Delta \xi$	0	0	0	0	0	-	0	-	0	-	0	-	

Table 5: Impacts Effects of Shocks on Key Endogenous Variables (Data and Model Comparison)

except for current account. The failure of DSGE models to replicate volatility is a pervasive phenomenon in the DSGE literature and our model is not immune to this problem yet.

	$\operatorname{sd}(c)$	$\operatorname{sd}(x)$	$\operatorname{sd}(y_h)$	$\operatorname{sd}(\pi)$	$\operatorname{sd}(l)$	$\operatorname{sd}(i)$	$\operatorname{sd}(\Delta\xi)$	$\operatorname{sd}(nx)$	$\operatorname{sd}(p_f/p_h)$	$\operatorname{sd}(G)$
Data	0.02	0.10	0.02	0.03	0.04	0.02	0.06	0.10	0.10	0.05
Model	0.02	0.02	0.001	0.004	0.01	0.01	0.01	0.11	0.05	0.001

Table 6: Volatility (Actual and Model)

Table 7 reports the model variance decompositions of the six key macroeconomic aggregates. The fluctuations of all these variables are significantly governed by the IST shock. For example, about 62% of the variation of GDP is explained by IST shock while TFP plays a minor role. This is an important property of the model which stands sharply in contrast with extant Indian DSGE models. The IST shock determines the relative price of investment goods. A negative IST shock could deter investment by raising the relative price of investment. Parente and Prescott (2000) argue that nations may raise barriers to the efficient use of technology which could raise the relative price of investment. In the context of India, pervasive corruption could be interpreted as a negative IST shock which could deter investment. Thus the predominant role of IST in determining key macroeconomic aggregates is not surprising. Next to IST, the domestic monetary policy plays a key role in determining the fluctuations of the macroeconomic aggregates. Foreign interest rate also plays a role. Zero importance of fiscal policy variable is due to the absence of distortionary taxation in the model and thus may not be taken seriously.<sup>9</sup> It appears that foreign export demand plays a minor role in determining the macroeconomic fluctuations.

	$A_t$	$Z_{x,t}$	i	<i>i</i> *	G	$y_H^*$
$Y_H$	0.39	62.24	34.46	2.80	0	.02
c	0.19	47.85	48.08	3.84	0	.02
x	0.13	52.00	44.22	3.63	0	.01
l	3.81	52.41	37.60	2.56	0	1.17
$\pi$	0.03	54.06	42.58	3.29	0	.03
$\Delta \xi$	0.03	45.39	50.78	3.75	0	.03

Table 7: Variance Decomposition (per cent)

# 6 Conclusion

During the last decade, the New Keynesian Dynamic Stochastic General Equilibrium framework (DSGE) has become the cornerstone for modern macroeconomic analysis. Following this evolution of macroeconomics research, in NCAER we aim to build a small open economy DSGE model for the Indian economy. Motivation behind this endeavour is to adopt the new generation analytical toolbox for studying business cycles of the economy. This working paper is produced as a part of this process. The paper reviews the existing literature on the DSGE modelling for India, explores the comovements of the key macroeconomic variables from the annual data (1970 - 2010), and develops a baseline model to reproduce the stylised facts of business cycles. In the near future, we will operationalise the model by estimating the structural parameters, and validate it through alternative sets of simulations. In addition, several features of the Indian economy, which are omitted in our present model, would be incorporated, such as, dualistic labour market, limited asset market participation, and financial frictions. To make the model operational, the baseline model can be estimated by the Bayesian method of estimation. Overall, the DSGE model for India would serve the purpose of policy evaluation and macroeconomic forecasting in a comprehensive fashion.

<sup>&</sup>lt;sup>9</sup>In a future extension, distortionary taxes would be introduced in the model.

# 7 Acknowledgement

This particular work at National Council of Applied Economic Research (NCAER) is generously supported by the Think Tank Initiative, managed by the Canadian International Development Research Centre. We are grateful to Shashanka Bhide and Bornali Bhandari for their valuable comments. We are also thankful to the workshop participants of "Dynamic Stochastic General Equilibrium Modelling for Emerging Markets and Developing Economies", held in NCAER on 22nd September 2014. Ajaya Sahu, Manan Bhatia, and Youngdae Lee are acknowledged for their competent and timely research assistance.

# 8 Data Appendix

Several sources are used to put together all the macroeconomic variables required for our analysis. We have listed the variables with their respective sources in the following table.

Variables with Description	Sources
Private consumption expenditure (Base year: 2004-05; in crores)	National Accounts Statistics
Consumer price index (Base year: 2010)	Labour Bureau
Employment (by working hours, in crores)	National Accounts Statistics
Govt consumption expenditure (Base year: 2004-05; in crores)	National Accounts Statistics
GDP deflator	National Accounts Statistics
Nominal interest rate (Call Money Rate; in percentage)	Handbook of Statistics on Indian Economy
Private Investment (Base year: 2004-05; in crores)	National Accounts Statistics
Investment to GDP Ratio (in percentage)	World Development Indicator
Investment Specific Technology Shock	Estimated residuals from Capital accumulation
Total Factor Productivity	Estimated Solow residuals
Foreign interest rate (in percentage)	Database of St. Louise FRED
Capital accumulation (Base year: 2004-05; in crores)	National Accounts Statistics
Import (Base year: 2004-05; in crores)	National Accounts Statistics
Import to GDP Ratio (in percentage)	World Development Indicator
Nominal exchange rate	Handbook of Statistics on Indian Economy
Price of imported goods	Handbook of Statistics on Indian Economy
Foreign economy GDP Deflator	UN Statistics, National Accounts Section
Price of exported goods	Handbook of Statistics on Indian Economy
Total consumption expenditure (Base year: 2004-05; in crores)	National Accounts Statistics
Wholesale price index (Base year: 1993-94)	MOSPI
Export (Base year: 2004-05)	National Accounts Statistics
Export to GDP ratio (in percentage)	World Development Indicator
Real GDP at Factor cost (Base year: 2004-05; in crores)	National Accounts Statistics
Foreign real GDP at constant prices of 2005 (in crores)	UN Statistics, National Accounts Section

# References

- Anand, R., Peiris, S., and Saxegaard, M., 2010. An estimated model with macrofinancial linkages for India. *IMF Working Paper Series*, WP/10/21.
- [2] Backus, D., Kehoe, P., and Kydland, F., 1994. Dynamics of the trade balance and the terms of trade: the *J*-curve. *American Economic Review*, 84: 84-103.
- [3] Basu, P. and Thoenissen, C., 2011. International business cycles and the relative price of investment, *Canadian Journal of Economics*, 44 (2): 586-606
- [4] Baxter, M. and King, R., 1999. Measuring business cycle fluctuations: approximate band pass filter for economic time series. *Review of Economics and Statistics*, 81, 4, pp 575-593.
- [5] Benigno, P., 2009. Price stability with imperfect financial integration. Journal of Money, Credit and Banking, Vol. 41, Issue (1): 121 - 149.
- [6] Bhattacharya. R., and Patnaik, I., 2013. Credit constraints, productivity shocks and consumption volatility in emerging economies. *IMF Working Paper No. WP/13/120*.
- [7] Christiano, L. J., Eichenbaum, M., and Evans, C., L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of Political Economy*, University of Chicago Press, University of Chicago Press, vol. 113(1), pages 1-45, February.
- [8] Christiano L.J., and Fitzgerald, T., J., 2003. The band pass filter. International Economic Review, vol. 44(2), pp 435-465.
- [9] Gabriel, V., Levine, P., Pearlman, J., and Yang, B., 2011. An estimated DSGE model of the Indian economy", Working Papers, National Institute of Public Finance and Policy, 11/95.
- [10] Goyal, A., 2011. A general equilibrium open economy model for emerging markets: monetary policy with a dualistic labor market. *Economic Modelling*, Elsevier, vol. 28(3), pp. 1392-1404, May.
- [11] Heathcote, J., and Perri, F., 2002. Financial autrarky and international business cycles. *Journal of Monetary Economics*, vol. 49, pp 601-27.

- [12] Hodrick, R J. & Prescott, E.C., 1997. Postwar U.S. business cycles: an empirical investigation. Journal of Money, Credit and Banking, Blackwell Publishing, vol. 29(1), pages 1-16, February.
- [13] Kollmann, R., 2002. Monetary policy rules in the open economy: Effects on Welfare and Business Cycles, *Journal of Monetary Economics*, pp 989-1015.
- [14] Levine, P., and Pearlman, J., 2011. Monetary and fiscal policy in a DSGE model of India. Working Papers, National Institute of Public Finance and Policy, 11/96.
- [15] Parente, S., and Prescott, E., C., 2000. Barriers to Riches, MIT Press, August.